Level – 1	**Daily Tutorial Sheet - 6**

77.(D) Integrating by parts we get: Given integral $=(3x-1)\sin x - 3\int \sin x\, dx - (1-2x)\cos x - 2\int \cos x\, dx$

$= (3x-1)\sin x + 3\cos x - (1-2x)\cos x - 2\sin x = (3x-3)\sin x + (2+2x)\cos x$

78.(C) $I = \int \frac{\sin^2 x}{\cos^6 x} dx = \int \tan^2 x \sec^2 x \,.\, \sec^2 x\, dx = \int \tan^2 x \left(1+\tan^2 x\right) . \sec^2 x\, dx$

Put $\tan x = t$ to get: $I = \int t^2\left(t^2+1\right)dt = \frac{t^5}{5}+\frac{t^3}{3}+C = \frac{\tan^5 x}{5}+\frac{\tan^3 x}{3}+C$

79.(B) $3\cos x + 2\sin x = \ell\left(4\sin x + 5\cos x\right) + m\left(4\cos x - 5\sin x\right)$

$\Rightarrow \quad \ell = \frac{23}{41}$ and $m = \frac{2}{41} \quad \Rightarrow$ The given integral is

$\frac{23}{41}\int dx + \frac{2}{41}\int \frac{4\cos x - 5\sin x}{4\sin x + 5\cos x} dx = \frac{23x}{41} + \frac{2}{41}\log\left|4\sin x + 5\cos x\right| + C$

80.(A) Put $x-2=u$ to get $I = \int \frac{x\,dx}{x^2-4x+8} = \int \frac{u+2}{u^2+4}\, du$

$= \frac{1}{2}\int \frac{2u}{u^2+4} du + 2\int \frac{du}{u^2+4} = \frac{1}{2}\log\left(u^2+4\right) + \tan^{-1}\left(\frac{u}{2}\right) + C = \frac{1}{2}\log\left(x^2-4x+8\right) + \tan^{-1}\left(\frac{x-2}{2}\right) + C$

81.(C) $\int e^x \left(\frac{1}{x+2} + \ell n\left(x+2\right)\right) dx = e^x \ell n\left(x+2\right) + C$

82.(A) $I = \int \frac{dx}{\sin^6 x} = \int \operatorname{cosec}^4 x \cdot \operatorname{cosec}^2 x\, dx$ [Put $\cot x = t \quad \Rightarrow \quad -\operatorname{cosec}^2 x\, dx = dt$]

$= \int \left(\cot^2 x + 1\right)^2 \operatorname{cosec}^2 x\, dx = \int \left(t^2+1\right)^2 \left(-dt\right) = -\frac{t^5}{5} - 2\frac{t^3}{3} - t + C$ (where $t = \cot x$)

83.(A) $I = \int \sqrt{1+\sec x}\, dx = \int \frac{\sqrt{\sec^2 x - 1}}{\sqrt{\sec x - 1}}\, dx = \int \frac{\tan x}{\sqrt{\sec x - 1}} dx$

[Put $\sec x - 1 = t^2 \quad \Rightarrow \quad \sec x \tan x\, dx = 2t\, dt \quad \Rightarrow \quad \tan x\, dx = \frac{2t\, dt}{t^2+1}$]

$I = \int \frac{1}{t}\left(\frac{2t}{t^2+1}\right) dt = 2\tan^{-1}(t) + C = 2\tan^{-1}\left(\sqrt{\sec x - 1}\right) + C$

84.(B) $I = \int \frac{1}{\left[\left(x+a\right)^2 - a^2\right]^{3/2}}\, dx$ [Put $x + a = a\sec\theta \quad \Rightarrow \quad dx = a\sec\theta \cdot \tan\theta \cdot d\theta$]

$I = \int \frac{a\sec\theta\tan\theta}{a^3\left(\tan^2\theta\right)^{3/2}} d\theta = \frac{1}{a^2}\int \frac{1}{\cos\theta} . \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \frac{1}{a^2}\int \frac{\cos\theta}{\sin^2\theta} d\theta = \frac{1}{a^2} . \frac{-1}{\sin\theta} = -\frac{1}{a^2} . \frac{x+a}{\sqrt{x^2+2ax}} + C$

(Make a triangle for $\cos\theta = \frac{a}{x+a}$ and find $\sin\theta$ as $\frac{x+a}{\sqrt{x^2+2ax}}$)

85.(C) We know that $\int \frac{1}{\sqrt{1+x^2}} dx = \log\left(x + \sqrt{1+x^2}\right) \quad \therefore \quad \frac{d}{dx}\log\left(x+\sqrt{1+x^2}\right) = \frac{1}{\sqrt{\left(1+x^2\right)}} \quad \therefore$

$I = \int t\, dt = \frac{1}{t}t^2 = \frac{1}{2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$

86.(A) $I = \int \frac{2 - \sin x}{2\cos x + 3}\, dx$

$$I = \int \frac{2}{2\left[\cos^2(x/2) - \sin^2(x/2)\right] + 3}\, dx - \int \frac{\sin x\, dx}{(2\cos x + 3)}$$

$$= \int \frac{2\sec^2(x/2)\,dx}{2\left[1 - \tan^2(x/2)\right] + 3\left[1 + \tan^2(x/2)\right]} + \frac{1}{2}\int \frac{-2\sin x}{(2\cos x + 3)}\, dx$$

Put $\tan\left(\frac{x}{2}\right) = t$ in 1st and in 2nd put $2\cos x + 3 = t$

$$I = \int \frac{4\,dt}{5 + t^2}\, dx + \frac{1}{2}\log(2\cos x + 3) = 4 \cdot \frac{1}{\sqrt{(5)}}\tan^{-1}\frac{t}{\sqrt{(5)}} + \frac{1}{2}(2\cos x + 3) + C$$

$$= \frac{4}{\sqrt{5}}\tan^{-1}\left(\frac{1}{\sqrt{(5)}}\tan\frac{x}{2}\right) + \frac{1}{2}\log(2\cos x + 3) + C$$

87.(A) Let $f(x) = ax^2 + bx + c$. By given condition, we get, $c = -3,\ a + b + c = -3,\ 3(4a + 2b + c) = -3$

These given $a = 1, b = -1, c = -3$

$$\therefore \quad f(x) = x^2 - x - 3 = \frac{f(x)}{x^3 - 1} = \frac{x^2 - x - 3}{(x-1)(x^2 + x + 1)} = -\frac{1}{x-1} + \frac{2(x+1)}{x^2 + x + 1} \qquad \text{[Using partial fractions]}$$

$$\therefore \quad \int \frac{f(x)}{x^3 - 1}dx = \int -\frac{1}{x-1} + \frac{2x+1}{x^2 + x + 1} + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= -\log(x - 1) + \log(x^2 + x + 1) + \frac{2}{\sqrt{3}}\tan^{-1}\frac{2x + 1}{\sqrt{3}} + k$$

88.(A) $\int \frac{x^3 - 1}{x^3 + x}dx = \int \frac{x^3 + x - (x+1)}{x^3 + x}dx$

$$= \int\left[1 + \frac{x+1}{x(x^2+1)}\right]dx = \int\left(1 - \frac{1}{x} + \frac{x}{x^2+1} - \frac{1}{x^2+1}\right)dx = x - \log x + \frac{1}{2}\log(x^2 + 1) - \tan^{-1}x + c$$

89.(D) Put $x = a\tan^2\theta \quad \therefore\ dx = 2a\tan\theta\sec^2\theta\,d\theta \quad \therefore\ I = \int \sin^{-1}\sqrt{\left(\frac{a\tan^2\theta}{a\sec^2\theta}\right)} \cdot 2a\tan\theta\sec^2\theta\,d\theta$

$$= \int \sin^{-1}(\sin\theta)\cdot 2a\tan\theta\sec^2\theta\,d\theta = a\int \theta\left(2\tan\theta\sec^2\theta\right)d\theta$$

Integrate by parts

$$I = a\left[\theta\sec^2\theta - \int \sec^2\theta \cdot 1\,d\theta\right] + C \qquad \left[\int 2\tan\theta\sec^2\theta\,d\theta = \int 2\sec\theta(\sec\theta\tan\theta\,d\theta) = 2\frac{\sec^2\theta}{2}\right]$$

$$= a\left[\theta(1 + \tan^2\theta) - \tan\theta\right] = a\left[\left(1 + \frac{x}{a}\right)\tan^{-1}\sqrt{\left(\frac{x}{a}\right)} - \sqrt{\left(\frac{x}{a}\right)}\right] = (a + x)\tan^{-1}\sqrt{\left(\frac{x}{a}\right)} - \sqrt{ax} + C$$

90.(B) $f(x) = \int \frac{2\sin x - \sin 2x}{x^3},\ x \neq 0 \qquad \lim_{x \to 0} f'(x) = \frac{2\sin x}{x}\left(\frac{1 - \cos x}{x^2}\right) = 1$